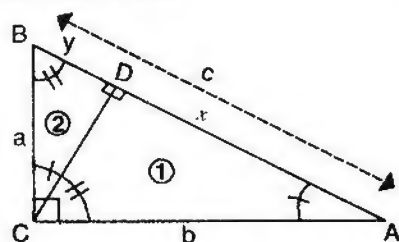


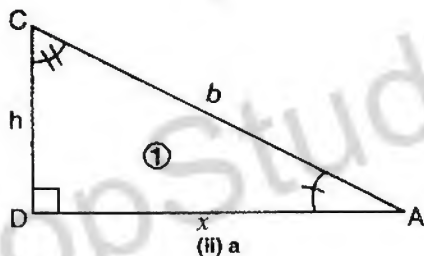
PYTHAGORAS THEOREM

Pythagoras Theorem

In a right angled triangle, the square of the length of hypotenuse is equal to the sum of the squares of the lengths of the other two sides.



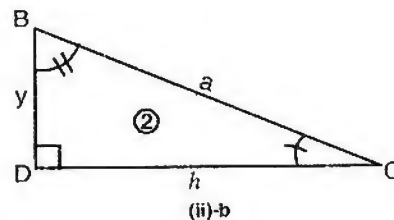
(ii)-a



(ii)-b

Proof (Using similar Δs)

Statements	Reasons
In $\triangle ADC \longleftrightarrow \triangle ACB$	Refer to figure(ii)-a and (i)
$\angle A \cong \angle A$	Common – self congruent
$\angle ADC \cong \angle ACB$	Construction – given, each angle = 90°
$\angle C \cong \angle B$	$\angle C$ and $\angle B$, complements of $\angle A$.
$\therefore \triangle ADC \sim \triangle ACB$	Congruency of three angles
$\therefore \frac{x}{b} = \frac{b}{c}$	(Measures of corresponding sides of similar triangles are proportional)
or $x = \frac{b^2}{c}$	
.....(i)	



(ii)-b

Given

$\triangle ACB$ is a right angled triangle in which $m\angle C = 90^\circ$ and $m\overline{BC} = a$, $m\overline{AC} = b$ and $m\overline{AB} = c$.

To Prove

$$c^2 = a^2 + b^2$$

Construction

Draw \overline{CD} perpendicular from C on \overline{AB} .

Let $m\overline{CD} = h$, $m\overline{AD} = x$ and $m\overline{BD} = y$.

Line segment CD splits $\triangle ABC$ into two \triangle s ADC and BDC which are separately shown in the figures (ii)-a and (ii)-b respectively.

Again in $\triangle BDC \leftrightarrow \triangle BCA$

$$\angle B \cong \angle B$$

$$\angle BDC \cong \angle BCA$$

$$\angle C \cong \angle A$$

$$\therefore \triangle BDC \sim \triangle BCA$$

$$\therefore \frac{y}{a} = \frac{a}{c}$$

$$\text{or } y = \frac{a^2}{c} \quad \dots\dots\dots(ii)$$

$$\text{But } y + x = c$$

$$\therefore \frac{a^2}{c} + \frac{b^2}{c} = c$$

$$\text{or } a^2 + b^2 = c^2$$

$$\text{i.e., } c^2 = a^2 + b^2$$

Refer to figure (ii)-b and (i)

Common-self congruent

Construction - given, each angle = 90°

$\angle C$ and $\angle A$, complements of $\angle B$

Congruency of three angles.

(Corresponding sides of similar triangles are proportional).

Supposition.

By (i) and (ii)

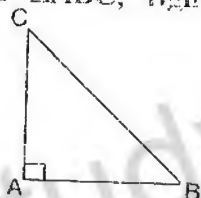
Multiplying both sides by c .

Corollary

In a right angled $\triangle ABC$, right angled at A .

$$(i) \overline{AB}^2 = \overline{BC}^2 - \overline{CA}^2$$

$$(ii) \overline{AC}^2 = \overline{BC}^2 - \overline{AB}^2$$

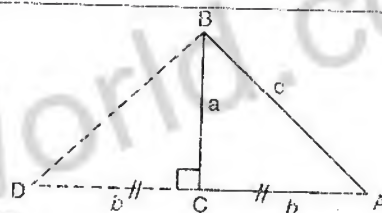


Converse of Pythagoras' Theorem

If the square of one side of a triangle is equal to the sum of the squares of the other two sides then the triangle is a right angled triangle.

Proof

Statements	Reasons
$\triangle DCB$ is a right -angled triangle.	Construction
$\therefore (\overline{BD})^2 = a^2 + b^2$	Pythagoras theorem
But $a^2 + b^2 = c^2$	Given
$\therefore (\overline{BD})^2 = c^2$	Taking square root of both sides.
or $\overline{BD} = c$	
Now in $\triangle DCB \leftrightarrow \triangle ACB$	
$\overline{CD} \cong \overline{CA}$	Construction



Given In a $\triangle ABC$, $\overline{AB} = c$, $\overline{BC} = a$ and $\overline{AC} = b$ such that $a^2 + b^2 = c^2$.

To Prove $\triangle ACB$ is a right angled triangle.

Construction Draw \overline{CD} perpendicular to \overline{BC} such that $\overline{CD} \cong \overline{CA}$. Join the points B and D .

$\overline{BC} \cong \overline{BC}$ $\overline{DB} \cong \overline{AB}$ $\therefore \triangle DCB \cong \triangle ACB$ $\therefore \angle DCB \cong \angle ACB$ But $m\angle DCB = 90^\circ$ $\therefore m\angle ACB = 90^\circ$ Hence the $\triangle ACB$ is a right-angled triangle.	Common Each side = c. $S.S.S. \cong S.S.S.$ (Corresponding angles of congruent triangles) Construction
--	--

Corollary: Let c be the longest of the sides a, b and c of a triangle.

- If $a^2 + b^2 = c^2$, then the triangle is right.

- If $a^2 + b^2 > c^2$, then the triangle is acute.
- If $a^2 + b^2 < c^2$, then the triangle is obtuse.

Exercise 15

1. Verify that the \triangle s having the following measures of sides are right-angled.

(i) $a = 5$ cm, $b = 12$ cm, $c = 13$ cm

Ans. $(\text{Hyp})^2 = (\text{Perp.})^2 + (\text{Base})^2$
 $(13)^2 = (12)^2 + (5)^2$
 $169 = 144 + 25$
 $169 = 169$

\therefore The triangle is right angled.

(ii) $a = 1.5$ cm, $b = 2$ cm, $c = 2.5$ cm

Ans. $(\text{Hyp})^2 = (\text{Perp.})^2 + (\text{Base})^2$
 $(2.5)^2 = (1.5)^2 + (2)^2$
 $6.25 = 2.25 + 4$
 $6.25 = 6.25$

\therefore The triangle is right angled.

(iii) $a = 9$ cm, $b = 12$ cm, $c = 15$ cm

Ans. $(\text{Hyp})^2 = (\text{Perp.})^2 + (\text{Base})^2$
 $(15)^2 = (12)^2 + (9)^2$
 $225 = 144 + 81$
 $225 = 225$

\therefore The triangle is right angled.

(iv) $a = 16$ cm, $b = 30$ cm, $c = 34$ cm

Ans. $(\text{Hyp})^2 = (\text{Perp.})^2 + (\text{Base})^2$
 $(34)^2 = (30)^2 + (16)^2$
 $1156 = 900 + 256$

$$1156 = 1156$$

\therefore The triangle is right angled.

2. Verify that $a^2 + b^2$, $a^2 - b^2$ and $2ab$ are the measures of the sides of a right angled triangle where a and b are any two real numbers ($a > b$).

Ans. In right angle triangle.

$$H^2 = P^2 + B^2$$

$$(a^2 + b^2)^2 = a^4 + b^4 + 2a^2b^2 \dots\dots\dots(i)$$

$$(a^2 - b^2)^2 = a^4 + b^4 - 2a^2b^2 \dots\dots\dots(ii)$$

$$(2ab)^2 = 4a^2b^2 \dots\dots\dots(iii)$$

Adding (ii) and (iii) we get

$$(a^2 - b^2)^2 + (2ab)^2 = a^4 + b^4 - 2a^2b^2 + 4a^2b^2$$

$$= a^4 + b^4 + 2a^2b^2 \dots\dots\dots(iv)$$

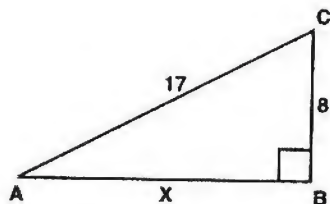
Comparing (i) and (iv), we get

$$(a^2 - b^2)^2 + (2ab)^2 = (a^2 + b^2)^2$$

Hence $a^2 + b^2$, $a^2 - b^2$ and $2ab$ are measures of the sides of a right angled triangle where $a^2 + b^2$ is Hypotenuse.

3. The three sides of a triangle are of measure 8, x and 17 respectively. For what value of x will it become base of a right angled triangle?

Ans:



Consider a right angled triangle

With $\overline{AB} = x$

$\overline{BC} = 8$

and $\overline{AC} = 17$

If x is the base of right angled $\triangle ABC$ then we know by Pythagoras theorem that

$$(\text{hyp})^2 = (\text{Base})^2 + (\text{perp})^2$$

$$(17)^2 = x^2 + (8)^2$$

$$289 = x^2 + 64$$

$$x^2 + 64 = 289$$

$$x^2 = 289 - 64$$

$$x^2 = 225$$

$$x = \sqrt{225}$$

As x is measure of side

So $x = 15$ units

Proof

Statements	Reasons
In right angled triangle	
$m\overline{CD} = 14\text{cm}$	$\overline{CD} = \frac{1}{2}m\overline{BC}$
$m\overline{AC} = 50\text{cm}$	Given
$(m\overline{AD})^2 = (m\overline{AC})^2 - (m\overline{CD})^2$	
$(m\overline{AD})^2 = (50)^2 - (14)^2$	$(m\overline{AC})^2 = (m\overline{AD})^2 - (m\overline{CD})^2$ (by Pythagoras theorem)
$= 2500 - 196$	
$= 2304$	
$\sqrt{(m\overline{AD})^2} = \sqrt{2304}$	
$m\overline{AD} = 18\text{cm}$	Taking square root of both sides
(ii) Area of $\triangle ABC = \frac{\text{Base} \times \text{Altitude}}{2}$	
$= \frac{28 \times 48}{2}$	
$= 14 \times 28$	
$= 672\text{ sq.cm}$	

4. In an isosceles \triangle , the base $\overline{BC} = 28$ cm, and $\overline{AB} = \overline{AC} = 50$ cm.

If $\overline{AD} \perp \overline{BC}$, then find:

(i) Length of \overline{AD}

(ii) Area of $\triangle ABC$

Given

$m\overline{AC} = m\overline{AB} = 50\text{cm}$

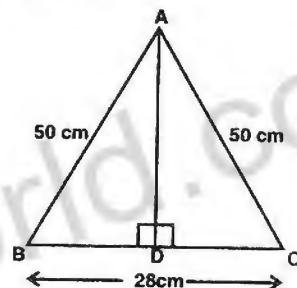
$m\overline{BC} = 28\text{cm}$

$\overline{AD} \perp \overline{BC}$

To Prove

$m\overline{AD} = ?$

Area of $\triangle ABC = ?$



In a quadrilateral ABCD, the diagonals \overline{AC} and \overline{BD} are perpendicular to each other.
Prove that:

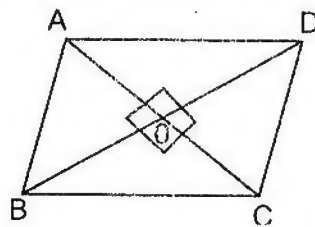
$$m\overline{AB}^2 + m\overline{CD}^2 = m\overline{AD}^2 + m\overline{BC}^2.$$

Given: Quadrilateral ABCD diagonal \overline{AC} and \overline{BD} are perpendicular to each other.

To Prove:

$$m(\overline{AB})^2 + m(\overline{CD})^2 = m(\overline{AD})^2 + m(\overline{BC})^2$$

Proof



Statements	Reasons
In right triangle AOB $m(\overline{AB})^2 = m(\overline{AO})^2 + m(\overline{OB})^2$(i)	By Pythagoras theorem
In right triangle COD $m(\overline{CD})^2 = m(\overline{OC})^2 + m(\overline{OD})^2$(ii)	By Pythagoras theorem
In right triangle AOD $m(\overline{AD})^2 = m(\overline{AO})^2 + m(\overline{OD})^2$(iii)	By Pythagoras theorem
In right triangle BOC $m(\overline{BC})^2 = m(\overline{OB})^2 + m(\overline{OC})^2$(iv)	By Pythagoras theorem
$m(\overline{AB})^2 + m(\overline{CD})^2 = m(\overline{AO})^2 + m(\overline{OB})^2 + m(\overline{OC})^2 + m(\overline{OD})^2$(v)	By adding (i) and (ii)
$m(\overline{AD})^2 + m(\overline{BC})^2 = m(\overline{AO})^2 + m(\overline{OD})^2 + m(\overline{OB})^2 + m(\overline{OC})^2$(vi)	By adding (iii) and (iv)
$(m\overline{AB})^2 + (m\overline{CD})^2 = (m\overline{BC})^2 + (m\overline{AD})^2$	By adding (v) and (vi)

6. (i) In the $\triangle ABC$ as shown in the figure, $m\angle ACB = 90^\circ$ and $\overline{CD} \perp \overline{AB}$. Find the lengths a , h and b if $m\overline{BD} = 5$ units and $m\overline{AD} = 7$ units.

Given: A $\triangle ABC$ as shown
 $m\angle ACB = 90^\circ$

and $\overline{CD} \perp \overline{AB}$

To prove : a , h and b .

In right angled $\triangle BDC$

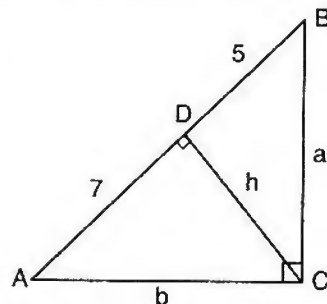
$$a^2 = 25 + h^2 \quad \dots\dots\dots (i)$$

in right angled $\triangle ADC$

$$b^2 = 49 + h^2 \quad \dots\dots\dots (ii)$$

in right angled $\triangle ABC$

$$a^2 + b^2 = 144 \quad \dots\dots\dots (iii)$$



adding (i) and (ii)

$$a^2 + b^2 = 74 + 2h^2 \quad \dots\dots\dots (iv)$$

from (iii) and (iv)

$$74 + 2h^2 = 144$$

$$2h^2 = 144 - 74$$

$$2h^2 = 70$$

$$h^2 = 35$$

$$h = \sqrt{35}$$

Now we will find a and b

Put $h^2 = 35$ (in Eq. 1)

$$a^2 = 25 + 35$$

$$a^2 = 60$$

$$a = \sqrt{60}$$

$$= \sqrt{4 \times 15}$$

$$a = 2\sqrt{15}$$

now put

$$h^2 = 35 \text{ (in Eq. 2)}$$

$$b^2 = 49 + 35$$

$$b^2 = 84$$

$$b = \sqrt{84}$$

$$b = \sqrt{4 \times 21}$$

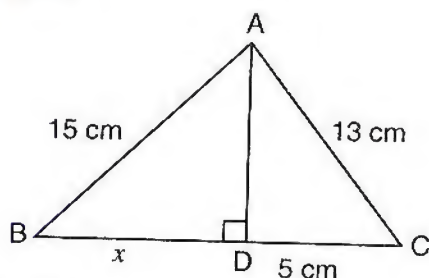
$$b = 2\sqrt{21}$$

SO $a = 2\sqrt{15}$

$$h = \sqrt{35}$$

$$b = 2\sqrt{21}$$

(ii) Find the value of x in the shown in the figure.



In right angled triangle ADC

$$m(\overline{AC})^2 = m(\overline{AD})^2 + m(\overline{DC})^2$$

$$(13)^2 = (\overline{AD})^2 + (5)^2$$

$$169 = (\overline{AD})^2 + 25$$

$$(\overline{AD})^2 = 169 - 25$$

$$(\overline{AD})^2 = 144$$

$$\overline{AD} = \sqrt{144}$$

$$\overline{AD} = 12 \text{ cm}$$

In right angled triangle ABD

$$(\overline{AB})^2 = (\overline{AD})^2 + (\overline{BD})^2$$

$$(15)^2 = (12)^2 + x^2$$

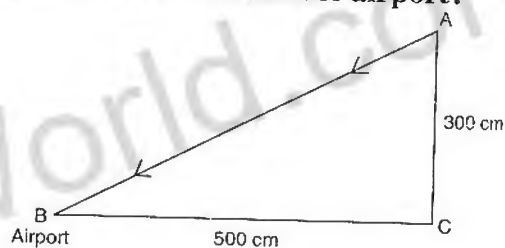
$$225 = 144 + x^2$$

$$x^2 = 225 - 144$$

$$x^2 = 81$$

$$x = 9 \text{ cm}$$

7. A plane is at a height of 300 m and is 500 m away from the airport as shown in the figure. How much distance will it travel to land at the airport?



Here A be the position of plane and B be the position of airport.

$$m\overline{AC} = 500 \text{ m}$$

$$m\overline{BC} = 300 \text{ m}$$

$$m\overline{AB} = ?$$

Applying Pythagoras theorem on right angled triangle ABC

$$|\overline{AB}|^2 = |\overline{AC}|^2 + |\overline{BC}|^2$$

$$= (500)^2 + (300)^2$$

$$= 250000 + 90000$$

$$= 340000$$

$$|\overline{AB}|^2 = 34 \times 10000$$

so $|\overline{AB}| = \sqrt{34 \times 10000}$

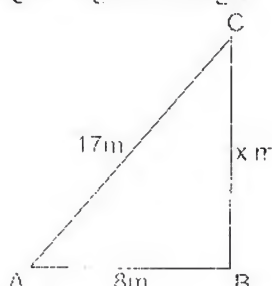
$$= \sqrt{34 \times 100 \times 100}$$

$$= 100\sqrt{34}m$$

So required distance is $100\sqrt{34}m$

8. A ladder 17 m long rests against a vertical wall. The foot of the ladder is 8m away from the base of the wall. How high up the wall will the ladder reach?

Ans. Let the height of ladder = x m
in right angled triangle



$$(\text{Hyp})^2 = (\text{Perp.})^2 + (\text{Base})^2$$

$$(17)^2 = (x)^2 + (8)^2$$

$$289 = x^2 + 64$$

$$x^2 = 289 - 64$$

$$x^2 = 225$$

$$x = \sqrt{225} = 15m$$

9. A student travels to his school by the route as shown in the figure. Find $m\overline{AD}$, the direct distance from his house to school.

According to figure, $m\overline{AB} = 2\text{km}$

$$m\overline{BC} = 6\text{km}$$

$$m\overline{CD} = 3\text{km}$$

Here $m\overline{AB}$ and $m\overline{CD}$ are perpendicular

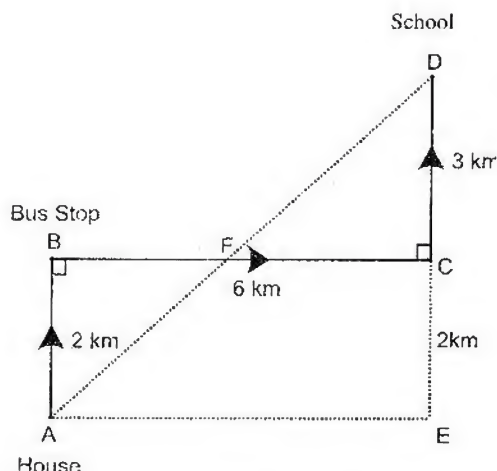
$$\text{Perpendicular} = \overline{AB} + \overline{CD}$$

$$= 2 + 3$$

$$= 5\text{km}$$

According to Pythagoras theorem

$$(H)^2 = P^2 + B^2$$



$$(m\overline{AD})^2 = (5)^2 + (6)^2 = 25 + 36$$

$$(m\overline{AD})^2 = 61$$

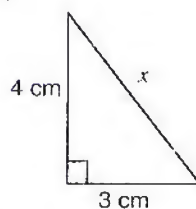
$$m\overline{AD} = \sqrt{61} \text{ Km}$$

10. Which of the following are true and which are false?

- (i) In a right angled triangle greater angle is 90° . (T)
- (ii) In a right angled triangle right angle is 60° . (F)
- (iii) In a right triangle hypotenuse is a side opposite to right angle. (T)
- (iv) If a, b, c are sides of right angled triangle with c as longer side then $c^2 = a^2 + b^2$. (T)
- (v) If 3 cm and 4 cm are two sides of a right angled triangle, then hypotenuse is 5 cm. (T)
- (vi) If hypotenuse of an isosceles right triangle is $\sqrt{2}$ cm then each of other side is of length 2 cm. (F)

11. Find the unknown value in each of the following figures.

(i)



By Pythagoras theorem

$$(\text{Hyp})^2 = (\text{Perp.})^2 + (\text{Base})^2$$

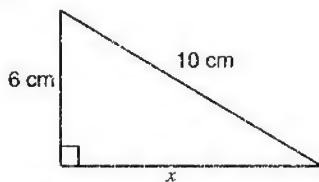
$$x^2 = (4)^2 + (3)^2$$

$$x^2 = 16 + 9$$

$$x^2 = 25 \Rightarrow x = \sqrt{25}$$

$$x = 5 \text{ cm}$$

(ii)



By Pythagoras theorem

$$(\text{Hyp})^2 = (\text{Perp.})^2 + (\text{Base})^2$$

$$(10)^2 = (6)^2 + (x)^2$$

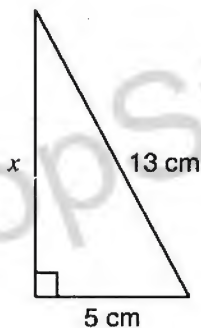
$$100 = 36 + x^2$$

$$x^2 = 64$$

$$x = \sqrt{64}$$

$$x = 8 \text{ cm}$$

(iii)



By Pythagoras theorem

$$(\text{Hyp})^2 = (\text{Perp.})^2 + (\text{Base})^2$$

$$(13)^2 = (x)^2 + (2)^2$$

$$169 = x^2 + 25$$

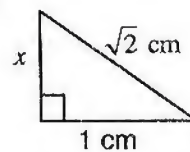
$$x^2 = 169 - 25$$

$$x^2 = 144$$

$$x = \sqrt{144}$$

$$x = 12 \text{ cm}$$

(iv)



By Pythagoras theorem

$$(\text{Hyp.})^2 = (\text{Perp.})^2 + (\text{Base})^2$$

$$(\sqrt{2})^2 = (x)^2 + (1)^2$$

$$(\sqrt{2})^2 = (x)^2 + (1)^2$$

$$2 = x^2 + 1$$

$$x^2 = 2 - 1$$

$$x^2 = 1$$

$$x = \sqrt{1} = 1 \text{ cm}$$

OBJECTIVE

- In a right angled triangle, the square of the length of hypotenuse is equal to the ____ of the squares of the lengths of the other two sides
 - Sum
 - Difference
 - Zero
 - None

- If the square of one side of a triangle is equal to the sum of the squares of the other two sides then the triangle is a ____ triangle.
 - Right angled
 - Acute angled
 - Obtuse angled
 - None

3. Let c be the longest of the sides a , b and c of a triangle. If $a^2 + b^2 = c^2$, then the triangle is ____:
- (a) Right
(b) Acute
(c) Obtuse
(d) None
4. Let c be the longest of the sides a , b and c of a triangle. If $a^2 + b^2 > c^2$, then triangle is:
- (a) Acute
(b) Right
(c) Obtuse
(d) None
5. Let c be the longest of the sides a , b and c of a triangle of $a^2 + b^2 < c^2$, then the triangle is:
- (a) Acute
(b) Right
(c) Obtuse
(d) None
6. If 3cm and 4cm are two sides of a right angled triangle, then hypotenuse is;
- (a) 5cm
(b) 3cm
(c) 4cm
(d) 2cm
7. In right triangle ____ is a side opposite to right angle.
- (a) Base
(b) Perpendicular
(c) Hypotenuse
(d) None

ANSWER KEY

1.	a	2.	a	3.	a	4.	a	5.	c
6.	a	7.	c						